The utility function and the emotional well-being function

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Abstract

Behind the utility function, which is the basis for economic and finance theory, is a philosophical and ethical approach based essentially on the Utilitarian and Hedonistic schools. Once qualitative, the utility function's approach shifted to a quantitative one based on the work of the mathematician, D. Bernoulli. This quantitative approach is normative and based on a maximizing agent. In this paper, the "emotional well-being" function is developed which mixes the ethics of a rational economic individual with those of a more complete individual who is simultaneously conditioned by economic, instinctive, social, religious, ethical, and esthetic values. This new utility function of emotional well-being is shown to remain within the envelope function of the Bernoulli-type utility function.

Keywords

Utilitarianism, utility function, rationality, well-being, ethics, emotional well-being

Introduction

Just as the utility function forms the basis of economic and finance theory, explicit normative definitions and assumptions form the basis of the utility function. The XVIII Century Utilitarian School was the primary philosophical backdrop for the utility function's development. The Utilitarian School itself was the product of schools of thought of many previous centuries. However, the normative nature of the utility function sometimes leads to a highly simplified analysis of individual economic behavior, whereas other aspects, such as the cardinal human virtues, including theological virtues, are omitted.

This paper presents the emotional well-being function in an attempt to explain outlying cases within the economy's normative context. Rather than invalidating the classical utility function theory, the emotional well-being function shows its predecessor to be an interpretation of a borderline mathematical solution. The emotional well-being function attempts to incorporate aspects of human virtues, the bases of ethics as behavioral sciences, into the analysis and to explain why individual decisions are not based exclusively on the rational economic standard, also known as the maximizer.

The paper is developed in two parts. The first part focuses on the underlying philosophical aspects of the utility function, its conceptualization, and some of its paradoxes. The second part is an analysis of the emotional well-being function, including its mathematical interpretation, its interpretation of behavior, ideas of risk, and a conclusion.

I. Utility function and utilitarianism

1.1. Underlying philosophical aspects of the utility function

The utility function which is used in finance theory is the result of an analytical, reflexive process stemming from ancient philosophical bases. Schumpeter (1954), in his classic work on economic thought, made a refined analysis of how utilitarianism, as a philosophical school, influenced the development of economic thought. The essence of the present-day utility function grew out of the concept of "useful", initially a purely ethical aspect. However, it would be an oversimplification to consider the utilitarianism of D. Hume (1711-1776), J. Bentham (1748-1832), and J. Stuart Mill (1806-1873) to be the utility function's only theoretical bases. Utility theory is also conceptually based on the Hedonism of Socrates, Plato, and Aristotle; in fact, the true origin of the present-day utility function lies within these schools.

As a philosophical and ethical school, Utilitarianism's ultimate objective is to maximize the positive effects associated with pleasure and to minimize pain. Pleasure is associated with happiness and pain, or the absence of pleasure, with unhappiness. In this sense, the primarily individual concept of "useful" can be taken to a social level, so that "useful" is that which provides the most happiness to the greatest number of individuals. Utilitarian thought, then, attempts to combine the pleasure and pain of different alternatives; in the course of its development, it even proposed calculations for pleasure and pain.

Hedonism also had a strong influence on the analytical construction of the present-day utility function. Hedonism, according to Epicurus, seeks the ethical objective of maximum pleasure, which is understood as that which is good but not extreme. Schumpeter (pg. 103 op. cit) referred to this scheme as a mechanistic philosophy of the universe, noting that the approach's social attitude is hedonism or very sublimated egocentric eudaemonism; Schumpeter considered J. Bentham to be a follower of the hedonistic line. Hedonism, its later schools, and Utilitarianism, then, conform the philosophical basis of the present-day utility function. It is important to remember that various other philosophical and ethical schools of thought have visions differing from those of utilitarianism.

Shifting from a qualitative concept of utility to a quantitative one allowed the formalization of the present-day utility function and the later deduction of marginal utility. Between 1730 and 1731, Bernoulli wrote the article where he presented his utilitarianism hypothesis. This contribution broadened the once-individual nature of utility to include a social level in which the degree of total satisfaction is determined to be the sum of individual satisfactions, making way for the following expression:

$$\sigma = \int_a^b k(dx/x) = k\ln(a/b)$$

where x = individual income; k = different individual proportionality factors; and a and b = satisfaction limits. Bernoulli's base was: dy = k(dx/x), where y = the "satisfaction" provided by income x, thereby giving rise to the first mathematical formulation of the utility function. In 1776, Quesnay developed the approach's economic conception. Thus, this formerly qualitative concept became the quantitative economic formulation of the utility function. This allowed the development of autonomous thought on the utility function, which, in its modern development, has grown into the ARA, RRA, and CRRA based on the work of Pratt-Arrow; Copeland and Weston (1992, pg. 85).

The normative principle of utilitarianism, and consequently, of the utility function, is maximum happiness. According to Schumpeter (op. cit. pg. 172), utilitarianism is a philosophy for real life, being both a normative system (with a strongly marked legal bias) and a social system. It is, then, a set of principles to be investigated, a working hypothesis. For Schumpeter, however, utilitarian hypotheses are completely lacking in value when it comes to matters of historical interpretation or the driving forces of economic history. He sees such hypotheses as being too weak to explain the aspects that escape economic behavior. In the field of economic theory, he adds, utilitarian hypotheses are useless, but not harmful.

The philosophical aspects of utilitarianism are also, by extension, those of the utility function. Moreover, they are fundamental to the understanding of what is behind quadratic or logarithmic utility functions; these kinds of utility functions are normally used, since they implicitly embody the utilitarian approach. Although the utilitarian approach is fundamental for understanding an investor's analysis of a given situation, which is an essential aspect in the field of finance, it lacks a vision of a complete being and, therefore, does not fully explain an individual making economic decisions. For our purposes, a complete individual will be defined as a biological being (with economic, instinctive, and erotic values), a social being (with vital and social values), a cultural being (with religious, ethical, logical, and esthetic values), and obviously as a "homo oeconomicus". The utility function fits all these values into one – that which maximizes the "useful", the individual's behavior thereby becoming that of a maximizer. In a strictly mathematical sense, the individual is then located over this utility function as can be seen in the upcoming point. From an economic and finance theory perspective, the utility function is a very good intellectual device which has simplified analysis, but is not able to give a good explanation of the sometimes problematic – and sometimes artificial – results of its application to policy.

1.2. Conceptualization of the utility function

Numerous economists have used Daniel Bernoulli's hypothesis in the development of the utility function theory. The mathematical influence on the description of the wealth versus utility phenomenon is, therefore, central to these definitions. Hence, the primarily mathematical conceptualization of the utility function.

So, let u (w) = utility function dependent on the level of wealth w.

The utility function and an individual's economic problem can be reduced to:

$$U(w) = \max_x E(e(w)), \text{ with } E(U/w) = \Sigma(p(w))w;$$

The above formula is the hypothesis of the expected utility of an individual's "rational behavior" when experiencing uncertainty. An individual having wealth w, the formula shows, can expect to maximize his or her utility or satisfaction level at some point in the future; this can have a probability of p(w). The question at hand is which distribution of wealth w will result in maximum utility. This utility function is, by definition, nondecreasing and bounded. According to Laffont (1995, pg. 8), two vital interpretations of the function should be highlighted:

a) First, it is necessary to accept that this definition is a working hypothesis and, therefore, requires the deduction of empirically verifiable implications. If these implications cannot be rejected on the basis of empirical work, then the conclusion reached is that people do act as if to maximize the expected utility. Laffont's observation, based on philosopher Carl Popper's method, is essential to the understanding and acceptance of the utility theory, and

b) Second, the normative interpretation, u(w), consists of demonstrating that rational agents "must maximize" their expected utility.

Rationality, therefore, is defined as the consistency of choosing among lotteries characterized by several axioms.

Both maximization and normativeness are essential aspects for understanding the implications of the development of the concept of a rational human under these assumptions. Any individual behaving in accordance with these two aspects is rational. A maximizer always prioritizes maximizing economic behavior (a related concept, optimizer, will be elaborated in the second part of this paper). Carroll (1998, pg. 14) develops the idea of maximizing economic behavior with Max Weber's (1958) definition of "the spirit of capitalism": the search for wealth for individual use and possession causes a capitalist system. Models and empirical verifications have been developed for this situation, as have alternatives, including those of Bakshi and Chen (1996) and Zou (1994). In another philosophical discussion on the topic, Joan Robinson (1962, pg. 17) says "utility maximization is a metaphysical concept of impregnable circularity".

In their classic work, Von Neumann and Morgenstern (1944) characterize rational agents with the following axioms: 1) the agent has complete preordering on the space M of lotteries, in other words, the agent is indifferent to the manner in which the consequences are obtained; 2) in every three preferentially ordered selections, the probability, α, is such that $\alpha a' + (1 - \alpha)a^3 \sim a^2$, in other words, there is continuity; and 3) the preferences are independent, that is, facing the same probabilities, the order of preferences does not change.

Debreu (1966) shows that the utility functions representing the behavior of rational agents are continuous and nondecreasing. Markowitz (1959) establishes the classical definitions of risk based on the utility function. Pratt (1964) and Arrow (1971) establish a classical measure of reward for risk, giving rise to the ARA (absolute risk aversion), RRA (relative risk aversion), and CRRA (constant relative risk aversion) coefficients.

The latter formed the basis for the development of "power utility functions", as indicated in Aït-Sahalia and Brandt (2001); Friend and Blume (1975); Kydland and Prescott (1982); Mehra-Prescott (1985); and Ang, Bekaert, and Liu (2000). One type of power utility function is from the following family:

$$u(w_{t+1}) = \begin{cases} \frac{w_{t+1}^{1-y}}{1-y} & \text{if } y > 1 \\ \ln(w_{t+1}) & \text{if } y = 1 \end{cases}$$

where "y" represents the RRA coefficient. This power utility function family has a constant CRRA and is popular in the treatment of financial asset portfolios.

More specific aspects are dealt with in the literature. One such case is that of ambiguous risk aversion, in which an agent faces an unknown return distribution. In this case, an agent may be unable or unwilling to assign probabilities to a return ensemble, for which Tversky and Kahneman (1992) propose the following utility function family:

$$u(w_{t+1}) = \begin{cases} -l(\bar{w} - w_{t+1})^b & \text{if } w_{t+1} < \bar{w} \\ (w_{t+1} - \bar{w})^b & \text{in other cases} \end{cases}$$

In the above model, the utility function is dependent on the relative earnings or losses resulting from investment strategies at the end of a given period. This proposition deals mostly with the problem of violated utility assumptions; individuals have a subjective tendency to favor results that are considered to be certain regarding income (or wealth), but which are, in reality, merely probable. This so-called "certainty effect" results in individuals working more losses or gains that are possible than with those that are certain.

Another aspect that has been incorporated into the utility functions is the willingness of agents to pay a minimum for information. Based on Bernoulli's utility function, Hwang and Satchell (2001) formulate how much of an agent's wealth should be used for the acquisition of information. Treich's definition of utility (1997) includes an explicitly assumed minimum payment, p_1, at the end of a given period that compensates for the investor's lack of information. The utility function is of the following type:

$$E_y(h(y)) = \max_x E\,[u(w_1 + p_1)]$$

Donations made by individuals have also received special attention. Literature exists which, excepting the assumptions of rational humans, has assimilated that different types of assets, including donations, behave like luxury assets. Carroll (2000) proposes adding a "joy of giving" to donation B(s) that would be given by the following family type:

$$B(s) = \frac{(s+y)^{1-\alpha}}{1-\alpha}$$

Carroll, Sidney, and Inhaber (1992) note that luxury assets are generally associated with wealth, such as arts, jewels, and sporting equipment. These items are always assets in an economic sense and, therefore, subject to the same pressures as any other economic asset. In another paper, Carroll (1998) says that, while love of wealth is certainly extreme as a motivation, it is not the only motivational force. A variety of other very plausible motivators exist, which appear to be every bit as motivating as the love of wealth. Some of these are job satisfaction, status, philanthropic ambitions, and power and they correspond to the complete individual referred to in the first part of this paper.

The first two parts of this paper outline the utility function's ancient philosophical bases as well as its strong normative component. The normative component assumes that individuals behave exclusively as maximizers and that only such behavior is rational. This last assumption implies that only one utility function is needed to explain an agent's behavior, and that this utility function is a good representation of any other motivation. This holds for functions – logarithmic, quadratic, continous, or twice differentiable – in which the investments are located exclusively and rationally on a curve's upward part (be it concave or convex), but never on the downward part. At the same time, the utility function can be used to explain the behavior of an economic agent at risk.

1.3. Paradoxes of the utility function

Schumpeter (op. cit. pg. 171) asks himself why this theory (utilitarianism) was so easily accepted by so many good brains. He then tells himself that it was because those good brains belonged to practical reformers who were battling a given historical situation which seemed, to them, to be "irrational" and in such debates, simplicity – even triviality – are the primary virtues of argumentation. He says: it isn't that those authors were insincere; we all are rapidly convinced of the nonsense we tend to preach.

It is paradoxical, then, that the utility function remains in use, only slightly modified from its original form, in spite of the reasonable critiques made of it. Von Mises (1968) helps to clarify this idea in his classic work "Human Action". He says that any interest in the action of an economic individual is only related to the individual as a participant in a market; the objective of the action does not matter. Furthermore, the nature of the action – be it altruistic or egotistical, rational or irrational – is irrelevant. Market action is information for the analysis of an economic problem; psychological information is the business of the psychologist; and social structures are problems for the sociologist.

Economics, as an autonomous science, needs its own definition of rationality in order to define an economic subject as a maximizer. Such a definition must be global in vision. Nevertheless, from a perspective of economic rationality, and using the utility function concept, it is difficult to explain the purely economic reasoning of the utility function when faced with varied investment situations. Following are some cases that are difficult to analyze by using the principle of economic rationality and that suggest the need to develop other hypotheses.

a) Charity and Donation. Non-profit institutions and donations as a form of financing have been studied economically and the use of utility functions in their analysis is noteworthy. Dixit and Stiglitz (1977) developed a utility function for the case, assuming price to be an endogenous variable and donations solutions for a market in equilibrium. Carroll's (2000 op. cit.) treatment of donations was also one of luxury assets, or rather, he explained donations as a simple matter of behavior when facing savings.

Parada (2001) showed that a donor's economic return (from a donation) tends to be a - ∞. This is, then, contrary to the assumptions of economic rationality. At the same time, non-profit institutions, which are normally financed with donations, can demand an economic return of -∞, also contrary to economic rationality. The exclusive use of economic rationality to explain donations and charity is, therefore, an extreme measure. What predominates, rather, is the vision of a complete individual. Thus, the economic rationality explanation is artificial for those donors who are inspired by values or ethical schools different from the utilitarianist school, including cultural, religious, spiritual, and "good Samaritan" donors.

It's hard for such donors to have their behavior explained by the utility function's implicit rationality, which is not to say, however, that there is not also an economic motive behind their donations. Simultaneous genetic, biological, social, and ethical factors can move an individual, even to the point of sacrificing something for the benefit of another. In other words, an individual may give up his or her maximum economic utility for an emotional sentiment that differs from purely economic rationality.

b) Ethical Investment Funds. Ethical and environmental investment funds have been in use since 1970. Analogous to a mutual fund portfolio, financial assets emitted by different production and service companies form this ethical and environmental portfolio. However, these funds are selected according to ethical and environmental criteria which are set by the financial intermediary and considered to be fundamental management aspects by the emitting companies. Purchasers are investors who are sensitized and concerned about ethics and the environment and, although they do seek returns, they are willing to sacrifice the maximum attainable returns offered by other funds for their principles – economic or otherwise. Sparkes (1995) demonstrated that, in the United States, ethical and environmental funds are also profitable.

In this case, the problem of the utility function can be simplified by using optimization with restrictions. This is, however, yet another utility function; mathematically, the restriction implies sacrificing the maximum. So it is that we arrive at a sub-optimum distanced from the basic principle of maximization.

c) Other cases. Some investors accept lower returns than those available in other feasible economic sectors in order to help decrease unemployment levels in high poverty areas; this could be a form of altruism. Others work in geographic zones dominated by the illicit drug economy without entering into this more lucrative business because of their moral and spiritual values. Although the utility function would explain this as a rejection of the risk involved in the business, such behavior could also be due to good intentions. Economically profitable interventions in the medical field, such as abortions, are often not selected as a means to income maximization because of moral and religious reasons. Patrons finance youth centers with reduced or null returns, for example, when they could obtain better economic returns elsewhere. The case of pledging credit is another example of work outside the maximum benefit zone. Although economic objectives are maintained, investments are located on an economic sub-optimum with regard to their potential (according to the utility function) and where benefits would be maximized.

The above cases clearly demonstrate that an individual's economic return can purposefully be less than the maximum indicated by the utility function. Hence, the hypothesis that other behavioral functions could exist, in which case, the utility function would be one member of a set. The distancing of an individual from the utility function's maximum benefit is normally assumed to be due to another, lower utility function within the same context of economic rationality.

How can the utility function be differentiated from other behavioral functions? This differentiation can be approached by means of the human virtues, cardinal as well as theological.

Human virtues and emotions, on which the scientific bibliography is abundant, are related to ethics and human behavior. They lead to the hypothesis that the economic utility function is the mathematical envelope function of another behavioral function capable of explaining the paradox of an individual acting on the downward part of the utility curve. Sharpe (1970) indicated that beyond point r*, where the quadratic utility function reaches its maximum, real utility decreases according to the increase of the rate of return. "This is," he pointed out "clearly unacceptable and perhaps this part of the curve should never be used for decisions when the income is over r*".

Unlike the normative perspective, the positive point of view finds the inability to explain the economic behavior of a return-seeking individual operating on the downward part of the curve to be too artificial and simplistic. It does not seem to be plausible to say that behavior not complying with the norm of the rational economic individual cannot be analyzed. Thus, the second part of this paper addresses the hypothesis of a greater function which, obviously, includes economic rationality.

II. Towards a function of emotional well-being

2.1. Mathematical interpretation

2.1.1. Propositions

Proposition 1. The Bernoulli-type logarithmic utility function is an envelope function for other curve families of the type: $BE(x) = a_1 sen(\pi x) + a_2 lnx + c_1$, with $a_1, a_2, c_1 > 0$.

Proof:

Let $U(x) = lnx + c$, Bernoulli-type utility function, where x = wealth. If BE(x)

has an envelope function, the parametric equations that determine said envelope function must satisfy the following conditions:

$$f(BE(x),\alpha)=0 \quad \text{and } f_\alpha(BE(x),\alpha)=0 \quad \text{where } \alpha = \pi x$$

In other words:

$$f(BE(x), x, \alpha) = a_1 sen(\pi x) + a_2 lnx + c_1 - BE(x) = 0$$

$$f_\alpha'(BE(x), x, \alpha) = a_1 x cos(\pi x) = 0$$

By solving, the envelope function is equal to:

$U(x) = (a_1 + c_1) + a_2 lnx$.

So we show that U(x), which is logarithmic, is an envelope function of another curve family.

Proposition 2: The function BE(x) has two envelope functions. The outer envelope function is given by the relative tangential maxima with U(x); the inner envelope function's points are tangential and minimum relative to the function BE(x). Between these two are U(x) functions differing only in the position of their coefficient C and intersecting with B(x) on the curve's downward part.

Proof:

Following the procedure for calculating the envelope function, the inner envelope functions of the type $U_2(x) = (-a_1 + c_1) + a_2 \ln x$, for which we have, in each tangential point, $dU_2(x)/dx = 0$ and $d^2U_2(x)/dx^2 > 0$ are deduced. In the outer envelope function, $U(x)_1$ is verified for each tangential point with BE(x) $dU_1(x)/dx = 0$ and $d^2U_1(x)/dx^2 < 0$; thus, these points are relative minima.

Let $U'(x) = a_2 \ln x + c'$; in x_2 with $x_1 < x_2$,

$U'(x_2) = BE(x_2)$, in other words, $a_2 \ln(x_2) + c' = a_1 sen(\pi x_2) + a_2 \ln x_2 + c$

So that $c' = a_1 sen(\pi x_2) + c$

$\therefore\ c' \neq c, \quad \forall\ x\ \varepsilon \{x \mid x \neq 1\},$ for which there is $U'(x)$.

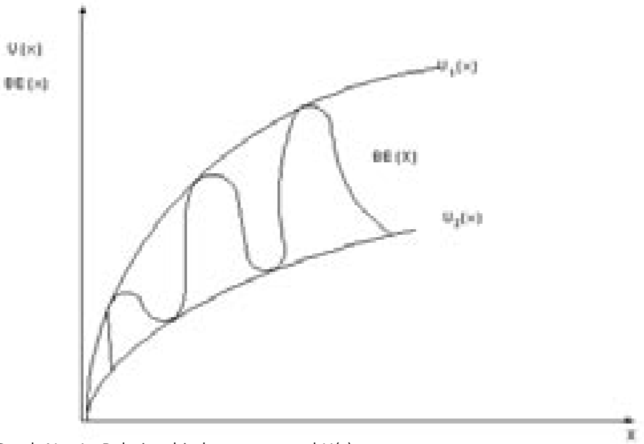
2.1.2. Characterization of the function BE(x)

Whether using the analytical method or visually inspecting the graph, BE(x), $U_1(x)$, and $U_2(x)$ are characterized by the following:

BE(x) is continuos in $[0,\infty]$ and twice differentiable; $\exists\ [z_i, z_j] \subseteq [0,\infty]$, so that in $[z_i, z_j]$, $i < j$, i and j $\varepsilon[0,\infty]$; BE(x) is ascending in $[z_i, x_k^*]$, which is concave and upward since ($BE'(x) > 0$ and $BE''(x) > 0$), and decreasing in $[x_k^*, z_j]$, where x^*_k are the relative maxima of BE(x)

$\forall k\ \varepsilon\ [0,\infty]$, $z_i < x^*_k < z_j$, and $\{z_i, z_j\}$ are the relative minima of BE(x). In x^*_k, it is known that $BE(x)^{max} = U_1(x)^{max}\ \ \forall x^*_k$, except x^*_1, where $U_1(x)^{max} > BE(x)^{max}$ depending on a_1, a_2, and c. In z_i and z_j we know that $BE(x)^{min} = U(x)^{min}$. Since $U_1(x)$ and $U_2(x)$ are on the curve's upward part in $[0,\infty] \rightarrow$ BE(x) is ascending in $[0,\infty]$, since $U_1(x)$ and $U_2(x)$ are the maximum and minimum envelope functions of BE(x) and, therefore, $BE(x_i) \geq 0\ \forall_i$, $i = 1, \infty$.

Graph No. 1 shows the situations of Propositions 1 and 2.



Graph No. 1. Relationship between x and U(x).

2.2. Interpretation of the emotional well-being function, BE(x)

2.2.1. Definitions and assumptions

- Individuals know their utility functions, U(x), which continue to fulfill all the normative aspects of the utility function theory.
- Individuals know that BE(x) and the envelope functions, $U_1(x)$ and $U_2(x)$, exist. They assume that their behavior is limited by these two U(x). $U_2(x)$ is the minimum emotional compensation required of a decision and $U_1(x)$ is the maximum emotional compensation possible.
- Emotional well-being is understood to be the degree of satisfaction resulting from an act, whether its motivation is purely economic rationality or a mixture of economic rationality, the cardinal virtues (prudence, justice, temperance, and bravery), and the theological virtues (faith, charity, and hope). The cardinal and theological virtues will be called the global ethic here in order to differentiate them from the rational economic ethic. In other words, the ethical approach is more global and includes religious values in conjunction with and simultaneous to the ethics of economic rationality, which can be individualized and separated in BE(x). All of this is captured by BE(x).
- It is assumed that emotional well-being, BE(x), can be represented by:

$BE(x) = a_1 sen\ (\pi x) + a_2 \ln x + c$, where x = wealth. (1)

- Let's assume that the coefficients a_1 and a_2 of BE(x) represent the relative weights that the individuals give to global ethics and economic ethics, respectively, with $a_1 + a_2 = 1$.

If $a_1 + a_2 = 1$, and if $a_2 = 1$, then: $U_1(x) = BE(x)$; that is, we are in the presence of a behavior guided solely by economic rationality ethics. On the other hand, if $a_1 = 1$; the individual will not give economic ethics more weight; in this case, the relevant ethics are the global ethics. An individual is expected to simultaneously use both economic rationality and global ethics, and where $a_1 = 1$ or $a_2 = 1$ are extreme cases. It should be noted, however, that the utility function theory U(x) is for the case where $a_2 = 1$.

- Economically, emotional compensations based on an emotional rationality stemming from global ethics are assumed to compensate the emotional sacrifices made by purposefully distancing oneself from the ethics of economic rationality.
- The level of wealth, x, can be interpreted as an interest rate, as shown by W. Sharpe (1970).
- Emotional well-being is assumed to depend on the level of wealth.

2.2.2. Analysis of the well-being function

The aforementioned definitions allow the following to be deduced:

$$\frac{dBE(x)}{dx} = a_1 \pi \cos(\pi x) + \frac{a_2}{x} \qquad (2)$$

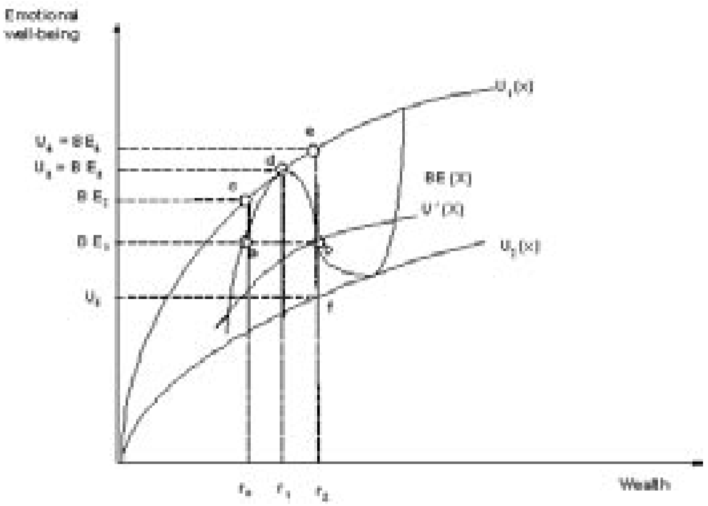
The first term on the right ride of equation (2) represents the impact of changes induced in emotional well-being exclusively by behavioral aspects (that is, $\pi a_1 \cos(\pi x)$) that do not correspond entirely to a rational economic individual. The influence of the value each individual grants to this behavior is represented as a_1. The second term represents the marginal contribution of emotional well-being to the individual's behavior as a rational economic being. Calculating, this is a_2/x, which is coincident with Bernoulli's classic approach, as cited in the previous pages. Note that, if $a_2 = 1$, then emotional well-being is completely explained by purely economic behavior. This means that the entire theoretical framework of the utility function is valid, since $BE(x) = U_1(x)$. It is interesting to note that, although the economic maximizing individual is presented as being borderline or extreme in the first part of this article, evidently such individuals do exist. There are cases, nevertheless, of individuals acting within certain utility margins without sacrificing their economic utility; in other words, $0 < a_2 < 1$.

Equation (2) also shows that, economically, the change in emotional well-being remains an inverse proportion of wealth to utility; Bernoulli's hypothesis is verified, in other words, since $d(B(x))/dx = a_2/x$. However, by conceiving of individuals as more than purely economic beings, others facets can be seen to have ascending or descending ranges in the change of emotional well-being. In this sinusoidally shaped logarithmic function, values can reach both the upward and downward parts of the curve. Intuitively, this suggests which variable results when including the analysis of values; in equation (2), this is represented by the first addend. Within the utility function, consideration is only given to individuals located on the upward part of the curve. Nonetheless, individuals can remain economic beings, retain their rational character, and still sacrifice utility for other goals. This is possible because the new interpretation of the economic being allows an individual to be located between the superior and inferior limits, on the curve's upward or downward part. This sinusoidal representation of the emotional well-being function makes the interpretation of human behavior more sensible and can explain the outcome of an individual's location on the downward part of said function.

The utility function, $U_2(x)$, indicates a minimum economic rationality and whether decisions are motivated by ethics, religion, spirituality, or some other value, there is an economic base below which the individual is not willing to make economic decisions. This analytical framework brings a slight change in the economic perception of risk. As will be seen later in this paper, the utility minimum defines the parameters of the economic sacrifice an individual is willing to accept in order to make a decision that is distanced from the economic rationality.

2.2.3. Joint analysis of BE(x) and U(x)

In the interest of simplification, the following analysis will be limited in focus to a part of the curve in Graph No. 2[1]:



Graph No. 2. The wealth – utility – emotional well-being relationship.

At level r_o, an emotional well-being of BE(r) is obtained. Economically, this should be $BE_1(r)$ of Graph No. 2 (point a). Therefore, although there is an economic sacrifice, the greater level of wealth, r_1, is still attractive. Obtaining an emotional well-being of $BE_3(r)$ on this point is coincident with the rationally economic utility of $U_3(x^*)$ of the graph. In other words, a total well-being is achieved here. From this point of view, the maximizer, an economic individual, need not analyze any function other than the rational economic individual's utility function, since $U_3(x^*) = BE(r^*)$ and r_1 is the point that maximizes the function BE(r).

A greater level of wealth, however, could accompany less economic utility because of the facets of human behavior differing from those of the rational economic individual. Here, wealth level r_2 obtains an emotional return of BE_1, or point b on Graph No. 2. This is equal to the benefit obtained in r_o. What might motivate someone to act this way? Here, the individual's behavior is motivated by values other than those indicated in the preceding paragraphs, such as the global ethic. The individual knowingly sacrifices the greater utility, point e, for ethical, religious, or spiritual reasons. What is sacrificed? Since an economic utility of U_4 (point e) can be obtained for wealth level r_2, the investor remaining in $BE_1(r_2) < U_4(r_2)$ makes an economic sacrifice of $U_4(r_2)-BE_1(r_2)$. The individual, nevertheless, still receives returns, as the minimum required is U_5 (point f). The compensation for an individual distanced from the economic sacrifice is:

$BE_1(r_1)-U_2(r_2)$. On the other hand, making this decision at point b results in another economic utility function, U'(x), which leaves the individual economically satisfied[2]. The individual, being complete, is willing to make an economic sacrifice in exchange for different kinds of compensation. Finally, a comparison between sections $U_4(r_2) - BE_1(r_2)$ and $BE_1(r_1) - U_2(r_2)$ gives the degree of the emotional well-being sacrifice or compensation.

The analysis of the new function, U'(x), is a response to the exclusively economic focus and is in accordance with the previously indicated human action approach of Von Mises (1968). In spite of its intellectual validity, the approach is not complete. The economic and value implications resulting from the $U_1(x)$ to U'(x) change should be explained through emotional well-being variables. Saying that individuals lower their expectations of economic demand, and that the reasons for doing so do not matter, seems to be a case of oversimplification. However, some people act this way because of both economics and values (related with the human virtues) and the latter cannot be abandoned as if they were neutral or constant when facing an economic decision.

In the downward part of the BE(x) function of Graph No. 2, the following situations are given between points b and d:

a.1. Sacrifice for not following the economic ethic = $U_4(r_2)$ - $BE(r_2)$. For small variations in wealth this is: $d_1U(x) = (a_2/x)dx$. Evidently, if the variations are large, then they require $\Delta U(x)$.

a.2. Decrease of the emotional well-being functions, given by:

$U_3 - BE_1 = \Delta BE(x) = [a_2 \ln(r_1) + (c_1 + a_1)] - BE(r_2)$

If Δx is small, then this decrease is:

$dBE(x) = (a_1 \pi \cos(\pi x) + a_2/x)dx$

a.3. In point $BE^*(x)$, $U^*_1(x) > BE^*(x)$ can be given, originating an initial loss that is carried to the analysis of the change of wealth in r_2, given by $U_1^*(r_1) - BE(r_1)^{max}$. The sacrifice of not behaving like an economic rational individual and accepting other value-related aspects that take the individual to point b of Graph No. 2 is:

(a.1) + (a.2) $\forall x^*_i$, $i = 2, \infty$. For i = 1, it can be (a.1) + (a.2) + (a.3). In other words, (a.1) is not earned, emotional well-being decreases from point d to b of Graph No. 2, and a probable loss in the first curve, or a total sacrifice = $(a_2/x)dx + (a_1\pi \cos(\pi x) + (a_2/x))dx$ is incurred.

In the downward part of BE(x), between points b and f, or between BE(x) and $U_2(x)$, the following situations are given:

a.4. On moving to a new utility function, a greater compensation is perceived which is given by:

U'(x)/dx. This is for the move from point $(r_1, U'_1(x))$ to $(r_2, U'_1(x))$. It is a new emotional economic situation.

a.5. A remnant of $U'(r_2) – U_2(r_2)$ represents well-being operating over a desired minimum. Satisfaction, which is given by $U'(r_2) - U_2(r_2)$, decomposes from two aspects: the new satisfaction of the economic individual for being above the new utility function U'(x) and a safety buffer. Both compensate the individual's economic loss for not having followed a purely economic ethic. This economic sacrifice has, at the same time, compensations allowing its BE(x) to be lower than the economic individual's maximization. Why do individuals distance themselves from the economic maximum to arrive at $BE(x) < U_1(x)$? Other facets of the complete individual explain this behavior, which an exclusively normative assumption of the rational economic individual cannot. The individual can also operate as an optimizer (and not a maximizer), being positioned within two ranges of economic utility, in other words, between $U_1(x)$ and $U_2(x)$. Finally, the individual will compare (a.1 + a.2) versus (a.3 + a.4).

2.3. On the subject of risk

The development of utility function theory regarding the evaluation and behavior of an individual facing risk is solid according to the quantitative evaluations, built on statistical measures and definitions, which are used to represent risk, as explained in Part I of this paper. Both Bernoulli-type functions and finance and economic theory's choice of investment opportunities are founded on the essence of this definition of risk.

Risk, then, is not a trivial topic in the development of an emotional well-being function. This is especially so because of contributions from other scientific areas, such as genetics and the study of attitude when facing risk (see Bishop and Waldholz, 1990, Ch. XV) and ethics in the form of the social sciences. The development of the BE(x) function has centered on the development of the human virtues which, although qualitatively defined, have an implicit concept of risk behind them. Human virtues, therefore, are difficult to measure with indices or statistical averages. Prudence, strength, temperance, and justice are the cardinal virtues on which the analysis concentrates, as indicated in Parada (2003).

Prudence encourages action with good intentions. It means being realistic, sensible, and correct. This first value obviously affects the degree of risk assumed. Strength is bravery in action, a sort of middle ground between fear (the rejection of risk) and daring (the "risk lover"). The meaning of economic utility theory, thus, is only completely relevant to the human virtue of strength. Temperance indicates moderation in mitigating the tendency toward excesses and, finally, justice operates as the behavioral standard of doing what is right. These four virtues, all value-related and qualitative, directly influence the level of risk assumed by a given individual.

Example:

If we suppose that an individual behaves as if BE(x) is given by $a_1 = 0.6$; $a_2 = 0.4$; and $c = 2$, then the analysis functions are:

$U_1(x) = 0.4 \ln x + 2.6$

$U_2(x) = 0.4 \ln x + 1.4$

$BE(x) = 0.6 \text{ sen } (\pi x) + 0.4 \ln x + 2$

With this data, and for $0.5 < x < 1.5$ (for the sake of simplification, only the first part of BE(x) is taken), we arrive at the following calculation:

Max(BE(X)) = 2.3660 con $x_o = 0.6126$ (according to Newton's numerical calculating method).

Supposing that we want to analyze an investor who is located on the downward part of the curve, BE(x), in $x_1 = 0.62$. This implies $\Delta x = 0.0074$. With this data, we arrive at the following:

	$x_o = 0.6126$	$x_1 = 0.62$	Δx
$U_1(x)$	2.404	2.4088	0.0048
$U_2(x)$	1.204	1.2088	0.0048
BE(x)	2.3668	2.3667	0.0001

This is the analysis of sacrifice and compensation for an investor located on the downward part of BE(x).

1. Emotional sacrifice for not being a rational economic individual.

a) Losses due to lack of economic rationality:

$dU_1(x) = (a_2/x)dx = (0.4/0.6126) \times 0.0074 \approx 0.005$

or $\Delta U = U_1(0.62) - U_1(0.6126) = 2.409 - 2.404 \approx 0.005$

b) Decrease in BE(x) due to location below the initial optimum:

$dBE(x) = (0.6\pi\cos(\pi x) + 0.4/x)dx$

$= 0.6\pi\cos(0.6126) + 0.4/0.6126)0.007 \approx 0$

or $BE(0.62) - BE(0.6126) \approx 0$

c) Initially (in x_o), we have $BE(0.6126) < U_1(0.6126)$; therefore we arrive at a sacrifice of: $U_1(0.6126) - BE(0.6126) = 0.0373$

So the total sacrifice is: $0.005 + 0.0373 = 0.04$

or $U_1(0.62) - BE(0.62) = 2.409 - 2.3667 \approx 0.0423$

2. Emotional compensation, or $BE(0.62) - U_2(0.62)$, for locating oneself higher than $U_2(0.62)$.

a) Compensation due to the new utility function, or U'(x):

$dU'(x) = (a_2/x)dx = (0.4/0.6126)0.0074 \approx 0.005$

b) Compensation due to being over the required minimum, $U_2(x)$:

$U'(0.6126) - U_2(0.62)$

This requires knowing the C' of U'(x).

Since $U'(0.62) = BE(0.62)$, then $0.4\ln 0.62 + C' = 0.6\text{sen}(0.62\pi) + 0.4\ln 0.62 + 2$

Or, $C' = 2.5579$

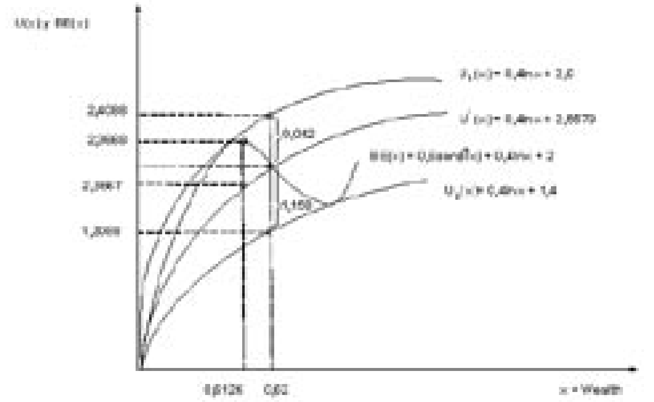
So $U'(0.6126) - U_2(0.62) = 2.3619 - 1.209 \approx 1.153$

Total emotional compensation = $0.005 + 1.1531 \approx 1.158$

or, $BE(0.62) - U_2(0.62) \approx 1.158$

In summary, this individual's position, which weights global ethical reasons at 60% and economic reasons at 40%, is on the downward part of function BE(x), below the economic utility function. If acting exclusively according to the economic ethic ($a_2 = 1$), the individual's satisfaction would equal $U_1(x) = 2.4088$. However, the individual opts for the lower satisfaction of $BE(x) = 2.3667$.

This creates an emotional sacrifice of 0.0423, but an economic satisfaction of 1.158. The investor can, then, operate on the downward part of BE(x), as Graph No. 3 shows:



Graph No. 3. Wealth-Emotional Well-being Relationship.

Conclusion

The process of developing the utility function theory, the basis of economic and finance theory, has been a long one. Initially influenced by Greek philosophers, the utility function eventually took shape after growing out of the ethical schools of Utilitarianism and Hedonism with the development of economics as an autonomous science. The central assumption is that the function is the response to an individual's behavior who, normatively, is a maximizer of utilities considering the level of wealth possessed. The utility function's explanation of individual behavior, however, falls short when its normative aspect is violated by the assumption that individuals are complete, and not exclusively economic, beings.

Therefore, a hypothesis develops of other families of functions, with the utility function being a borderline or extreme solution (mathematically, the outer envelope function, with another utility function serving as the inner envelope function). A new function, called emotional well-being, develops between these envelope functions. Emotional well-being describes a behavioral mix between the rational economic individual and the individual who does not follow this norm.

The newly developed function gives a framework in which to understand this behavior, from which, obviously, the functioning of purely economic rationality does not escape. Nevertheless, purely economic rationality is always implicit in the analysis, since the actions are carried out between a band of two economic utility functions. The function shows emotional well-being to lie between a maximum that coincides with the economic maximum and a minimum level of emotional acceptance defined as a satisfaction minimum.

[1] In the functions BE(x), $U_1(x)$, and $U_2(x)$, the axes can be moved so that they pass through the origin, which is more economically sensible. This implies that the x values in the functions can be $x = x' + 1$, since if $x = 0 \rightarrow$ emotional well-being also equals zero.

[2] This new function is: $U(x) = a_2 \ln x + c'$. Since $U'(x)$ goes through point $BE(r_2) \Rightarrow U'(r_2) = BE(r_2)$. Replacing, one arrives at $c' = a_1 \text{sen}(\pi x) + c$.
If $x = 1 \Rightarrow c' = c$.

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